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Synchronizing Hyperchaotic Maps to Encode/Decode Information

Sincronización entre Mapas Hipercaóticos para Codificar y Decodificar Información

Carlos Aguilar-Ibáñez, Miguel S. Suárez-Castañón. Humberto Sossa-Azuela and Ricardo Barrón-Fernández
Centro de Investigación en Computación del IPN
Av. Juan de Dios Bátiz s/n Esq. con Manuel Othón de Mendizabal Col. San Pedro Zacatenco, A.P. 75476 07700 México, D.F., México
caguilar@pollux.cic.ipn.mx, sasuarez@prodigy.net.mx
Phone: 52-5-7296000, x-56568

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Abstract

In this work we propose to use hyperchaotic maps synchronization to encode and decode information. The information to be encode is input to the transmitter as an external perturbation. The transmitted signal is used for synchronization and as the encode information carrier. Once the receiver is synchronized with the transmitter, the former decode the information by reconstruct the external perturbation. Roughly speaking, we design a simple schema to encode and decode data, as a simple inverse problem approach. The schema performance shows to be quite satisfactory, as assess from the numerical implementation. We use the results to build an application to establish secure on-line communication over Internet.
Keywords Information Encoding, Information Decoding, Cryptography, Hyperchaotic, Map Synchronization

Resumen

En este trabajo se propone el uso de sincronización entre mapas hipercaóticos para codificar y decodificar información. La información a ser codificada es introducida al transmisor como una perturbación externa. La señal transmitida es empleada tanto para la sincronización y como portadora de la información codificada. Una vez que el receptor esta sincronizado con el transmisor, el primero decodifica la información mediante la reconstrucción de la perturbación externa. En términos generales, se diseñó un esquema sencillo para codificar y decodificar datos, enfocado como un problema inverso. El desempeño del esquema mostró ser muy satisfactorio, como se comprobó en la implantación numérica. Los resultados obtenidos se usaron para construir una aplicación para comunicación segura en línea sobre internet.
Palabras Clave: Información Codificada, información decodificada, criptografía, hipercaótico, Sincronización de Mapas

1 Introduction

One of the major advantages that Internet offers is the facility to establish a communication channel. However, the secrecy of the exchanged information is usually compromised. One way to protect information is by using the advantages that computer security offers. One of the main issues that computer security tries to accomplish is to ensure that sensitive information, specially that one moving across Internet, cannot be seen by unauthorized people. Perhaps the most powerful tools available to accomplish this task are those provided by Cryptography (see [Schneier, 1995], [Stinson, 2002], [Goldreich, 2000]). The purpose of Cryptography is to modify the content of some piece of information by means of an algorithm and some secrete information known as the cryptographic key, such that it could be almost impossible or not practical for an intruder to recover it.

Cryptography is not the only way to secure sensitive information. As we mentioned before, while cryptography allows us for concealing the content of messages, the goal of steganography is to conceal their existence. The basic idea is to embed some message onto a cover message. Maybe the most common technique exploits the fact that it is possible to modify some of the less significant bits of each byte of an image file in order to hide a message and keep the image almost equal to the original one. The original image (cover message) is the container, and the image file obtained after applying the steganographic process is the stego-object.

In this work we introduce an alternative option to enforce the secrecy of exchanged information through Internet, by means of nonlinear exact state reconstructors (see [Sira *et al.*, 2002]) of hyperchaotic maps (see [Carrol, 1998]) and chaotic system synchronization (see [Ulrich *et al.*, 1999] and [Ulrich, 1999]), as a tool to recover encoded messages in a

discrete-time chaotic signal. Chaotic system synchronization provides with the possibilities of encoding information using one or more state variables of a chaotic system as a carrier in the side of the sender, and decode it on the side of the receiver. This idea to our knowledge has been not used to solve this problem. The main idea is at least to confuse an intruder exploiting the chaotic nature of the state variables used as a carriers and hide the information moved from one point to another (see [Cuomo *et al.*, 1997]).

The scheme here presented is based on the synchronization of an encoding system (chaotic system), say the sender, and a decoding system (exact state reconstructor), namely the receiver. For synchronization we must understand that under a signal transmission, the receiver is able to reconstruct (maybe asymptotically) that of the sender. The system synchronization problem has been extensibility studied in general in (see [Special Issue, 1997]), and particularly their application to chaotic systems synchronization has been founded in (see [Pecora, 1990] and [Huijberts *et al.*, 2000]). Previously works, have described the use of synchronization of chaotic systems in the transmission of encoded information, using some state variables for synchronization and different state variables of the same system for encoding that information. Our work differs, because we use the same state variable for synchronization and encoding, which is less expensive in terms of communication channels and computation effort. The methology we use in order to carried out the synchronization process consist on applying the delay embedding method introduced in nonlinear time series based on the Takens' Theorem (see [Takens, 1981], [Parlitz *et al.*, 1994], [Packard, 1980], [Sauer *et al.*, 1991] and [Itoh *et al.*, 1997]).

Finally, it is worth mentioning that our proposal does not pretend to substitute the modern cryptographic and steganographic techniques. We strongly believe that our proposal is an alternative that could be more efficient in some situations, taking into account that the computational effort required is much less that the one needed for instance in Cryptography, ours as we will see is much faster. On the other hand, in spite that the security level of our scheme is based on the chaotic nature of the system that we use, we can not say that at the moment that it provides a higher degree of security.

The remaining of this work is organized as follows. Section II covers a brief description of chaotic volume-preservative-maps. Section III is devoted to probe that the chaotic maps presented in previous section are constructible with respect to a suitable output. Section IV shows the results obtained by numerical simulations. For this, we present two examples where our proposal is used to encode-decode information. Section V describes an actual application that we built to communicate over Internet using encoded messages by means of the results presented in this work. We give several examples to illustrate this. In short, Section VI is devoted to some conclusions and suggestions for ongoing research.

2 Background on Chaotic Maps:

Preliminaries

Let us consider a class of nonlinear iterative map, defined as:

$$X(k+1) = F(X(k)) \tag{2.1}$$

where $X(k) \in \Re^n$ given by $X(k) = [x_1(k), x_2(k)..., x_n(k)]^T$ and $F(\cdot)$ is a nonlinear iterative map given by $F(X(k)) = [x_2(k), x_3(k),..., M(X(k))]^T$, with

$$M(X(k)) = \left\{ \sum_{i=1}^{i=n} a_i x_i(k) + j \right\} \bmod 2j - j .$$

$\{a_1,...a_n\}$ is a set of fixed constants and $k = \{1,2,..., N\}$. Function $\{f + j\} \bmod 2j - j$, means that we take f and add the integer j, divided by $2j$, we keep the remainder, and subtract two for each j. In some cases $M(X(k))$ can also be replaced by the continuous function:

$$M(X(k)) = \sin\left(\sum_{i=1}^{i=n} a_i x_i(k) \right),$$

because as in the case of the *modulus* function, for any point x_i of the *sin* function that goes beyond some bounded region, the function moves it back into that region.

Notice that the characteristic polynomial of the linearized system is given by $p(s) = a_1 + a_2 s + ... + a_n s^n$.

This kind of systems exhibits a chaotic behavior in the cases where the Jacobian of the map in (2.1) around the equilibrium point zero has one or more eigenvalues outside the unitary circle (see [Carrol, 1998], [Pecora *et al.*, 1997] and [Lichtenberg, 1983]). It is desirable that the Jacobian of this map has two or more roots outside the unitary circle. If this is the case, their behavior is hyperchaotic. Another very important characteristic of this kind of systems, is that the values of their state variables are always confined to a *n*-dimensional hypercube, no matter what the initial conditions are. If the initial conditions are outside of the hypercube, eventually for some *n-th* iteration the state vector will be inside the hypercube.

It is important to say that hyperchaotic maps have desirable properties that help to accomplish most of the needs that arise in any private and secure communication systems. These maps do not present defined patterns or attractors; they can be synchronized in a few steps and they can be easily implemented in circuitry. In fact the mechanism that is described in the next section only needs *n-1* steps to synchronize for a *n*-dimensional system.

Let us now consider the slight modified version of the system given by (2.1) as follows:

$$\begin{aligned} X(k+1) &= F(X(k)) + Bw(k) \\ y(k) &= h(X(k)) = x_1(k) \end{aligned} \qquad (2.2)$$

where $B \in \Re^n$ is a constant vector defined by $B = \begin{bmatrix} 0 & 0 & \dots & \lambda \end{bmatrix}^T$ with $0 < |\lambda| << 1$; $y(k)$ and $w(k) \in R$, are the output and the external perturbation, respectively, and $F(\cdot)$ is the nonlinear map previously defined.

Based on the class of systems described in (2.2) we propose a simple mechanism to encode/decode information under the following assumptions:

A.1) At each *k*-iteration, we dispose of the output values $\{y(k-n),...,y(k-1),y(k)\}$, for all $k > n$.

A.2) The external perturbation satisfies the following: $w_k = 0$, for, $k \le n$ and $w_k \neq 0$ for $k > n$.

A.3) The set of parameters $\{a_1,...a_n\}$ are selected such that the characteristic polynomial has one or more roots outside the unitary circle. The initial conditions must be selected in such way that the system does not present periodical behavior.

3 Dead-Beat Synchronization

Let us consider the nonlinear iterative chaotic map given by (2.1). Then, this class of systems has two important properties:

i) The observability map Ψ defined by

ii) $\Psi = \left[h(k), h \circ F(x),..., h \circ F^{n-1}(x)\right] = X(k)$ satisfies that the Jacobian $\partial\Psi / \partial X$ equals the identity matrix, therefore the system (2.1) is called strongly locally observable around $x = 0$.

iii) It has unique equilibrium point which is given by $x_1(k) = x_2(k) = ... = x_n(k) = x_e$. Indeed from (2.1) we have two possibilities:

$$x_e = \left\{ \left(\sum_{i=1}^{i=n} a_i \right) x_e + j \right\} \bmod 2j - j$$

or

$$x_e = \sin\left(\left(\sum_{i=1}^{i=n} a_i \right) x_e \right)$$

which evidently has unique solution $x_e = 0$.

Hence, according to **Proposition 2.1** described in [Sira *et al.*, 2002] (see Appendix), system (2.1) is constructible with respect to the output $y(k)$ (for more details refer to [Isidori, 1989] and [Kotta, 1995]). Then, a map $\Psi : \Re^n \to \Re^n$ exists, such that state $X(k)$ can be exactly reconstructed from time $k = 0$, on terms of the output $y(k)$ and a finite string of previously obtained output, in the form: $X(k) = \varphi\big(y((k), y(k-1),..., y(k-n-1)\big),\ k \geq 0$.

Notice that the above relation is according to the well known Takens' Reconstruction Theorem (see: [Takens, 1981] and [Sauer *et al.*, 1991]).

The following proposition provides the main result which allows us to reconstruct the external perturbation $w(k)$ of (2.2)

Proposition 1 Let us consider the system given by (2.2), under the assumptions A.1 and A.2, then the following estimator recovers the external perturbation **w**, *delayed **n**-steps with respect to the output **y**, as follows:*

$$\hat{w}(k-n) = y(k) - \left\{ \sum_{i=1}^{n} a_i y(k-(1+n)+i) + j \right\} \bmod 2j - j \qquad (3.1)$$

Proof:

From (2.1), we clearly have that:

$$\begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+n-1) \end{bmatrix} = \begin{bmatrix} x_2(k-1) \\ x_3(k-1) \\ \vdots \\ \left(\sum_{i=1}^{i=n} a_i x_i(k-1) + j \right) \bmod 2j - j + w(k-1) \end{bmatrix} \qquad (3.2)$$

From the above vectorial equation, we note that

$$x_i(k-1) = y(k+i-1) \qquad (3.3)$$

Thus, the last component of the vectorial equation (3.2), can be expressed as:

$$x_n = \left\{ \sum_{i=1}^{i=n} a_i y(k+i-2) + j \right\} \bmod 2j - j + w(k+1) \qquad (3.4)$$

From (3.3) we have that $x_n(k+1-n) = y(k)$. By substituting this result into (3.4), we have, after some algebraic manipulations, the external perturbation *w* delayed *n*-steps with respect to the output *y(k)*, as follows:

$$\hat{w}(k-n) = y(k) - \left\{ \sum_{i=1}^{i=n} a_i y(k-(1+n)+i+j) \right\} \bmod 2j - j \qquad (3.5)$$

then clearly from (3) and (6), we obtain:

$$\left| \hat{w}(k-n) - k(k-n) \right| = 0 \ for k > n \qquad \bullet$$

□

Based on Proposition 1, we have designed a practical scheme to encode/decode information and, from this scheme we have developed an actual Internet application that allows to securely communicate across Internet. The degree of confidence of our scheme relies on the chaotic or hyperchaotic nature of the carrier signal and the difficulty to exactly reproduce it without the knowledge of the initial conditions and the parameter values used during the encoding process.

Note that in the schema proposed in [Sira *et al.*,2002] the transmitter has to send two or more signals to the receiver in order to allow the later to decode the message. At least one signal contains the needed key to reconstruct the remaining states of the transmitter system. The other signals are the encoded messages.

On the other hand the schema presented in this article, only has to send one signal, which simultaneously is used as a carrier of the message and the key needed to recover that message, that was input as an external or hexogen perturbation into the transmitter system.

4 Applications to Recovering Encoded Information Embedded into a Chaotic Signal

Numerical Simulations

Let the following nonlinear system act as an encoding system:

$$\begin{aligned} x_{k+1} &= z_k \\ z_{k+1} &= y_k \\ y_{k+1} &= 5\sin(ax_k + bz_k + cy_k) \end{aligned} \qquad (4.1)$$

and let w_k^1 and w_k^2 the messages to be masked, defined as follows: $w_k^1 = (\cos(i/2)\sin(i/10))/10$ and $w_k^2 = abs(\sin(2\pi i/100))$ where $i = 1,2,...,100$.

If we want to mask message w_k^1 with the output of the encoding system (4.1), then we have: $u'_{k+1} = 5\sin(ax_k + by_k) + \lambda w_k$.

Figure 1 shows graphically w_k^1 and the behavior of y_k and u'_k.

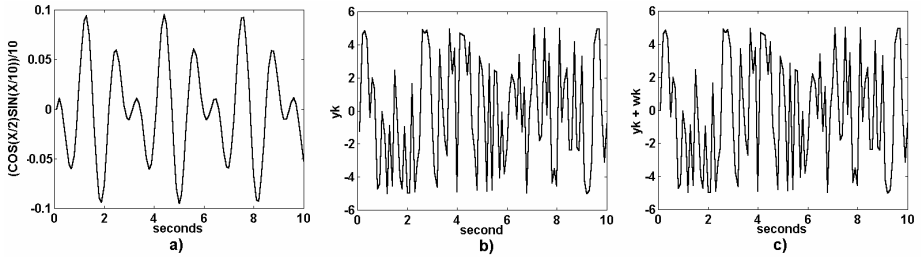

Fig. 1. a) Message, b) Chaotic Signal, c) Encoded Message with $\lambda = 0.1$

Figure 2 shows graphically the behavior of the chaotic signal and the encoded message, where now the message to be masked is w_k^2. We can see that the results shown in Figures 1 and 2 look alike.

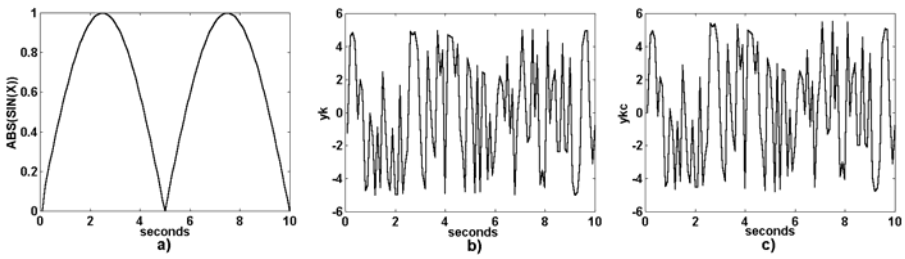

Fig. 2. a) Message, b) Chaotic signal, c) Encoded Message with $\lambda = 0.5$

Figure 3 shows the recovered signals (Figures. 1(a) and 2(a)).

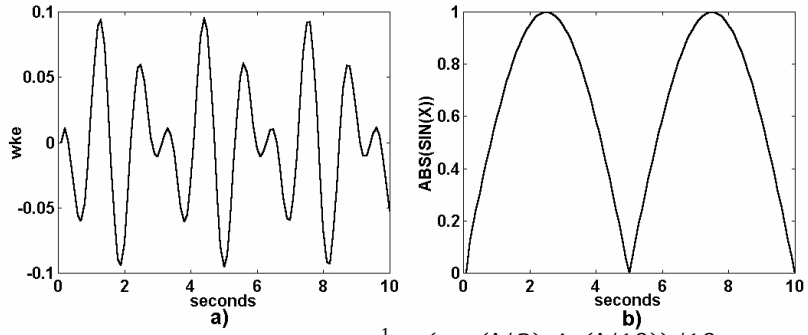

Fig. 3. Recovered Signals: a) $w_k^1 = (\cos(i/2)\sin(i/10))/10$
b) $w_k^2 = abs(\sin(2\pi i/100))$

Figure 4 shows the results obtained when applying the encoded mechanism, using different values for λ. Note the resemblance among them.

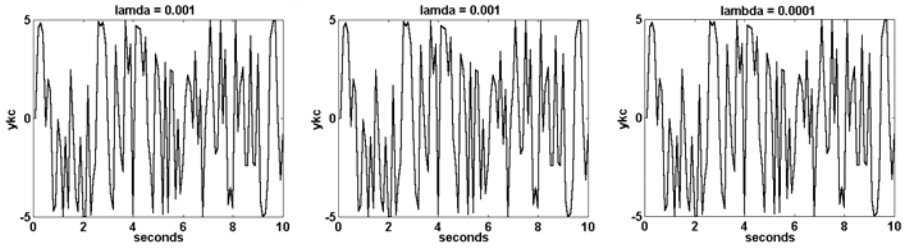

Fig. 4. Encoding Message containing signal $w_k^1 = (\cos(i/2)\sin(i/10))/10$

An Encoding Mechanism

Taking into account **Proposition 1**, we describe in this section an encoding mechanism, which uses the chaotic output of the system given by (2.1), and the message to be masked as the perturbation $w(k)$. If $w(k)$ is much smaller than $F(X(k))$, then $F(X(k)) + w(k) \approx F(X(k))$, thus the difference between $(F(X(k)))$ and $(F(X(k)) + w(k))$ is minimal and the chaotic or hyperchaotic nature of $F(X(k))$ is kept in $F(X(k)) + w(k)$.

Now, according to **Proposition 1**, we can estimate $w(k)$, that is, we can decode the masked message.

We present now our main result: a numerical encoding-decoding mechanism. In the sequel, DCED-Mechanism will stand for Discrete Chaotic Encoding-Decoding Mechanism.

DCEC-Mechanism.

1. *We send to an authorized recipient the output of the discrete-time chaotic system, i.e., the encoded message* (u_k'). *The authorized recipient is the person who possesses the exact state reconstructor and the needed information to decode the encoded message.*
2. *The recipient reconstructs the chaotic signal, i.e.,* (y), *then she or he performs the following operation:* $w_k = \dfrac{(u_k' - y_k)}{\lambda}$, *which allows to decode the encoded message.*

In the next section we give a description of the application that we developed to communicate across Internet using the previous results obtained in this work.

5 An Internet Application: Secure Communication Using an Insecure Communication Channel

The popularity of Internet as an efficient resource to communicate people and to move information from one site to another, brings the necessity of mechanisms to ensure the integrity of the data flowing across that world wide net. It also needs to avoid that non-authorized people have access to it. As we mentioned in the first section, the most used tools to ensure information secrecy are those that modern cryptography provide, and maybe in a more modest degree those derived from steganographics mechanisms.

In this section we describe an application we build using the results obtained in this work. This system allows to communicate on-line with other people across the Internet as we usually do when we use, for instance, the so-called chat rooms. The whole conversation will be however encoded, and even if somebody can see the bytes that we send or receive, she (or he) won't be able to figure out our conversation.

The system is organized in four modules or subprograms:

a) New Users Registry Module.

b) Start Session Module.

c) Users Logged Control Module.

d) Communication Module.

Figure 5 shows the block chart of the system.

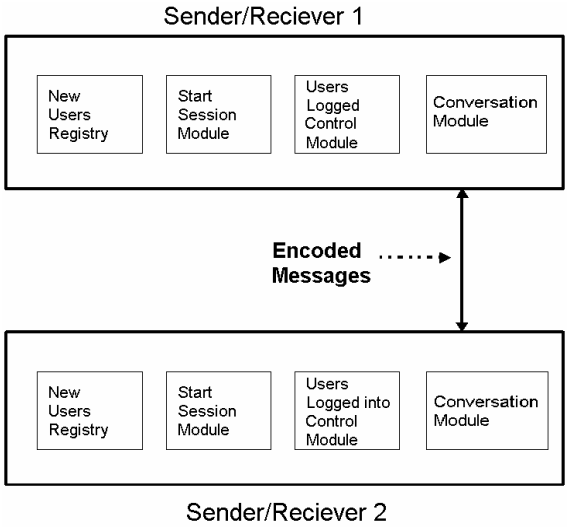

Fig. 5. Block Char of the Secure Communication System

Each module was implemented in Java™, version 1.3. Each module has its own graphical interface. The graphical interphase was build using the classes provided by the Package Swing, included in the SDK™ 1.3. Data manipulation was done using the MySQL™ data base manager. The modules needed to interact with the data base made use of the JDBC provided by MySQL™, and can be found in *www.mysql.com*. It is worth mentioning that the tools needed to build this system are freely available in the Internet.

5.1 New Users Registry Module

This module is composed of two submodules: the graphical interface module and the data base module. The graphical interface module provides a friendly interface that easily allows the user to insert her or his name into the user's data base. This is a necessary condition to be able to use the system, and to eventually communicate with the users that are already registered. The database module manages the whole information of each user, such as the user names and their passwords.

Fig. 6. This figure shows the graphical interface provided in the New Users Registry Module, that allows to register new users

5.2 Start Session Module

This module allows any already registered user to star a session and to communicate with other registered users. One of the main tasks of this module is to avoid that non-authorized users can log into the system.

Every time an user wants to communicate with another logged user, she (or he) must provide their user name and their password through out the provided graphical interface, shown in Figure 7. Once the module has the user name and the corresponding password, it provides access to the users' data base to allow or denied access to the system accordingly to the validation process result. Finally, when the user is logged into the system she (or he) will be able to contact any already logged user through the Users Logged Control Module, described below.

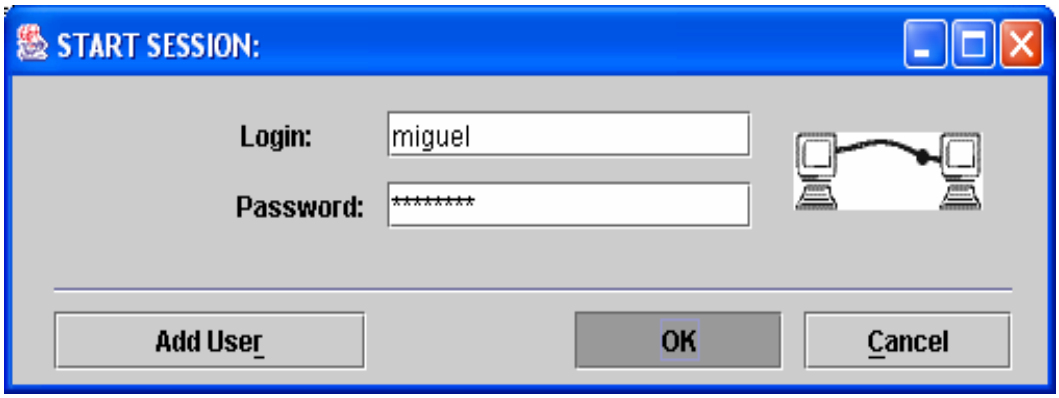

Fig. 7. This figure shows the graphical interface provided in the Start Session Module, that allows registered users to login into the system

5.3 Users Logged Control Module

This modules provides a list of logged users, by means of a graphical interphase. In order to establish a conversation, an user most double-click the username of the person who she (or he) wants to talk with.

This module works as follows. Every user logged to the system has his own communication channel or thread and is marked as connected into the data base; the module then checks the data base and refresh it into the graphical interface the list of users that are currently using the system. As mentioned before, when a user wants to talk with another connected user, she (or he) has to double-click their user name; then the module gets the IP address from the server data base. Once the module had the IP address, it makes a connection request to this address and if the receiver accepts the connection, a conversation will start. Figure 8 shows the graphical interface of this module.

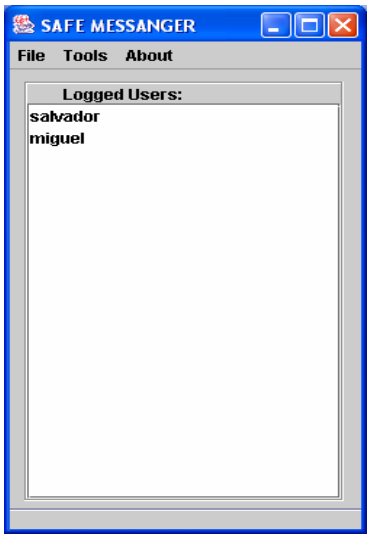

Fig. 8. This figure shows the graphical interface provided in the Users Logged Control Module, which shows two logged users

5.4 Conversation Module

This module hides and extracts the messages in each side of the conversation and provides the services needed to move information from one side to the other. The module consists of four submodules: Encoding Submodule, Decoding Submodule, Communication Submodule and Graphical Interface Submodule. The Encoding and Decoding Submodules as their names suggest, encode and decode the information to be sent and the information received, respectively. Both operation are executed accordingly to the results presented in Section III.

The Communication Submodule is needed basically to handle the transmission channel where the encoded information flows, it is also needed to refresh the list of logged users. Every logged user has an instance of this submodule. When an user doubles click the user name of another user a conversation request is made. If the user who receives the conversation request accepts it, then a threat between both users is created. The threat will handle every aspect of the communication among each pair of users. It is important to mention that each threat is an independent process, and it will allow any user to have more than one conversation in a simultaneous fashion. Figure 9 shows the Conversation Module graphical interface and tries to illustrate how two users (Ricardo and Salvador) carried out a typical conversation.

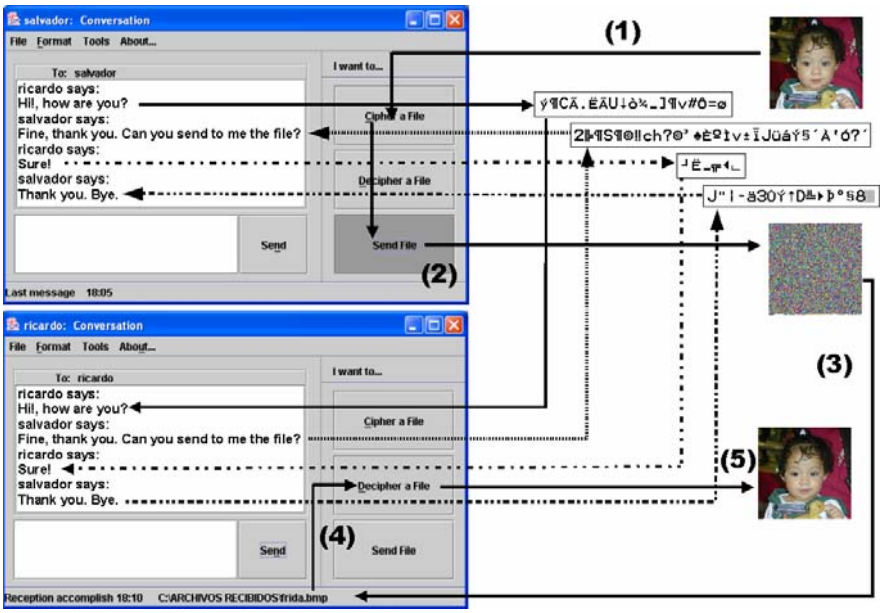

Fig. 9. This figure shows the graphical interface provided in the Communication Module. It also illustrates how a typical conversation is carried out

The image used in example of Figure 9 is a bitmap of 463 Kb. The time required to encode and decode was around 4 seconds in each case. Both process were done in a personal computer Dell™ Dimension DIM4300 with an Intel™ Pentiun™ at 1.6GHz and 128Mb in RAM, running Microsoft™ Windows XP Home Edition ver. 2002.

6 Conclusions

We have described in this work a kind of nonlinear maps that have the peculiarity of being chaotic or hyperchaotic accordingly to their eigenvalues. More important we have explained how to determine which behavior should be expected. We have also shown that it is possible to reconstruct the state vector of this n-dimensional maps if $(n-1)$ delayed values of the output are available. The reconstructor is designed based on the Takens' reconstruction theorem. This reconstruction totally differs from the traditional asymptotic synchronization approach which is based on nonlinear state observers', the common generated error is completely avoided. The obtained reconstruction results were applied to a modified version of the hyperchaotic maps originally presented, which include a scaled perturbation signal. After some algebraic manipulation it is clear that the system state vector and the applied perturbation are easily recovered. It is important to mention that the reconstructor only needs one signal to synchronize with the system, which in our case is also the carrier signal. The findings are illustrated with two examples.

An application (software) for secure communication over Internet using the results obtained was developed. This application allows to establish a secure communication channel between different sites. The user can communicate on-line or they can encode any computer file and then send it to a recipient who can decode it. To be able to access the application, any user must be registered in an users' database that is located in a public server. The system provides the facility to generate different encode/decode parameters to enforce the secrecy among users.

It is known that one of the major applications of chaotic systems lies in the secure communication area. Due to the properties of the nonlinear maps and their reconstructor described above, they are a suitable option in that area.

Appendix

Proposition 2.1 *Let the nonlinear chaotic system,* $x_{k+1} = f(x_k), y_k = h(x_k)$ *be locally observable, and suppose that corresponding to the constant value,* y_e*, there exists a unique state vector equilibrium value,* x_e*. Then, the system is* constructible, *i.e. there exists a map* $\varphi : \Re^n \to \Re^n$ *such that the state* x_k *of the system can be exactly reconstructed, from time* $k = 0$*, on, in terms of the output* y_k *and a finite string of previously obtained outputs, in the form:* $x_k = \varphi\left(y_k, y_{k-1}, ..., y_{k-(n-1)}\right), k \geq 0$ *provided the string of outputs,* $\{y_k\}$ *for* $y - n + 1 < k \leq 0$*, is completely known. Moreover, an initializations of (3.5) with arbitrarily chosen values,* $y_{-i}, i = 1,2,...,n-1$*, and the actual* y_0*, still results in an exact reconstruction of* x_k *for all* $k \geq n-1$

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Carlos F. Aguilar-Ibañez*. He was born in Tuxpan, Ver., México. He received the B.S. degree in Physics from the Higher School of Physics and Mathematics of the National Polytechnic Institute (Mexico City) in 1990. From the Research Center and Advanced Studies of the N.P.I (Mexico City), he received the M.S. degree in Electrical Engineering in 1994, and after postgraduate studies in Automatic Control, he received the Ph.D. degree in June of 1999. From that year until the present Dr. Aguilar has been at the Centre of Computing Research.*

Miguel Santiago Suárez. *He was born in México, City, México. He received the B.S. degree in Cybernetics and Computer Science from the School of Engineering of the University of La Salle, A.C. (México, City) in 1991. From the Research Institute of Applied Mathematics and Systems (México, City) in 2001 he received the M.S. degree in Computer Sciences. From 2000 until now he is a PhD student at the Center of Computing Research of the National Polytechnic Institute (Mexico City).*

Juan Humberto Sossa Azuela. *He received the Ph.D. Degree in Informatics in 1992 from the National Polytechnic Institute of Grenoble, France in 1992. He is currently a Professor of Computer Vision at the Center for Computing Research of the National Polytechnic Institute of Mexico. His research interests are in Image Processing, Computer Vision and Pattern Recognition, in particular invariant theory, image retrieval and associative memories, theory and applications.*

Ricardo Barrón Fernández. *He received the M.Sc. Degree in Computer Sciences in 2002 from the Center for Computing Research of the National Polytechnic Institute of Mexico, where he is a Professor of Neural Networks and currently pursuing a Ph.D. His research interests are in Image Processing, Computer Vision and Pattern Recognition, in particular dynamic systems and associative memories, theory and applications.*